Investigation of Model Sensitivities and Model Errors
With Relation to Data Assimilation Systems

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LONG-TERM GOALS

To investigate basic predictability issues related to model sensitivities to uncertainties in the initial and boundary conditions; how these sensitivities are dependent on different model dynamics and different data-types; how their assessment can lead to the evaluation of model errors as intrinsic component of the data assimilation system used.

OBJECTIVES

a) To assess the dynamical mechanisms that ultimately limit the predictability of a flow system.
b) To assess how the flow predictability is dependent upon uncertainties in the initial and/or boundary conditions.
c) To assess uncertainty in models by evaluating estimates of model error covariances from the assimilation method used.

APPROACH

The effort of this first year of work has been on investigating the predictability of strongly nonlinear, jet-like currents which can be used as idealized models for western boundary currents such as the Gulf Stream. An increasing interest and amount of research has been in fact devoted in recent years to the prediction of ocean currents, especially those associated with a rich and energetic mesoscale eddy field. For such nonlinear systems, the most important factor that ultimately limits its predictability is the stability of the circulation that is the object of the forecast in the presence of perturbations arising from errors or uncertainties in the model initial and boundary conditions.

Traditionally, the stability of dynamical flow fields has been investigated through the so-called normal model analysis, in which one searches for the growing perturbations, the normal modes of the system, characterized by an exponential growth rate. If no growing normal modes exist, the flow is assumed to be stable. These ideas have been challenged by Farrell (1990) and Farrell and Moore (1992) who have shown that perturbations to the flow can exist which are characterized by rapid transient growth and that such transient growth can occur in the absence of unstable normal modes. These optimal perturbations characterized by rapid transient growth over a prescribed time-interval τ are the so-called singular vectors of the model. They characterize the regions in the system which may rapidly lose predictability before the effects of nonlinearity become important. As shown by Moore (1991) and Moore and Farrell (1993) they can be found through the use of the adjoint of the dynamical model used. Specifically, the tangent linear form is derived of the dynamical model as well as its adjoint. If

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R(t_1,t_2) is the linear propagator of the tangent linear model in the time interval $\tau = t_2-t_1$, and R$^+$(t_1,t_2) is the adjoint of R, then the singular vectors, or so-called optimal perturbations, are the eigenvectors of

$$R^+(t_1,t_2) \: X \: R(t_1,t_2)$$

where X is a weight function (Moore and Mariana, 1999). We have followed this approach to investigate the stability, and hence predictability, of non-linear jet systems as prototypes of the Gulf Stream, characterized by propagating meanders and meander growth downstream often leading to the formation and detachment of Gulf Stream rings.

This work is part of a collaboration under the DRI between Dr. Malanotte-Rizzoli of MIT and Dr. Steven Meacham of AER. At MIT, Dr. Jim Lu became Post-doctoral Associate under the ONR-DRI Grant NØØØ14-98-1-0881 on 04/01/1999. At AER, Dr. Amala Mahadevan became Postdoctoral Associate under the ONR-DRI grant of Dr. Steven Meacham.

The major scientific objective is to investigate the stability and predictability of Gulf Stream-like jets in idealized configurations by applying optimal perturbation analysis and by comparing two different models endowed with different stability properties. At MIT, Dr. Lu has focussed on the use of a quasi-geostrophic, reduced gravity vorticity equation model for which he has developed the adjoint and evaluated the singular vectors. At AER, Dr. Mahadevan has carried out the same analysis but for the 2-layer version of the quasi-geostrophic model that is endowed with baroclinic instability. As an example, in Fig. 1 we show the streamfunction patterns obtained from the reduced-gravity model at different times in the cycle of energy vacillations. Specifically, the left panels correspond to minima in the energy cycle, i.e. low energy states characterized by a short penetration scale of the Gulf Stream Jet into the basin interior. The right panels correspond to maxima in the energy cycle, i.e. high energy states characterized by a long penetration scale of the jet. Fig. 2 shows the corresponding dominant singular vectors, the left panels showing the vectors corresponding to low energy states of the circulation, and the right panels the vectors corresponding to high energy states. The optimal perturbations characterizing the low energy states are concentrated near the western boundary, where the optimal perturbations characterizing the high energy states spread from the boundary into the interior along the jet axis.
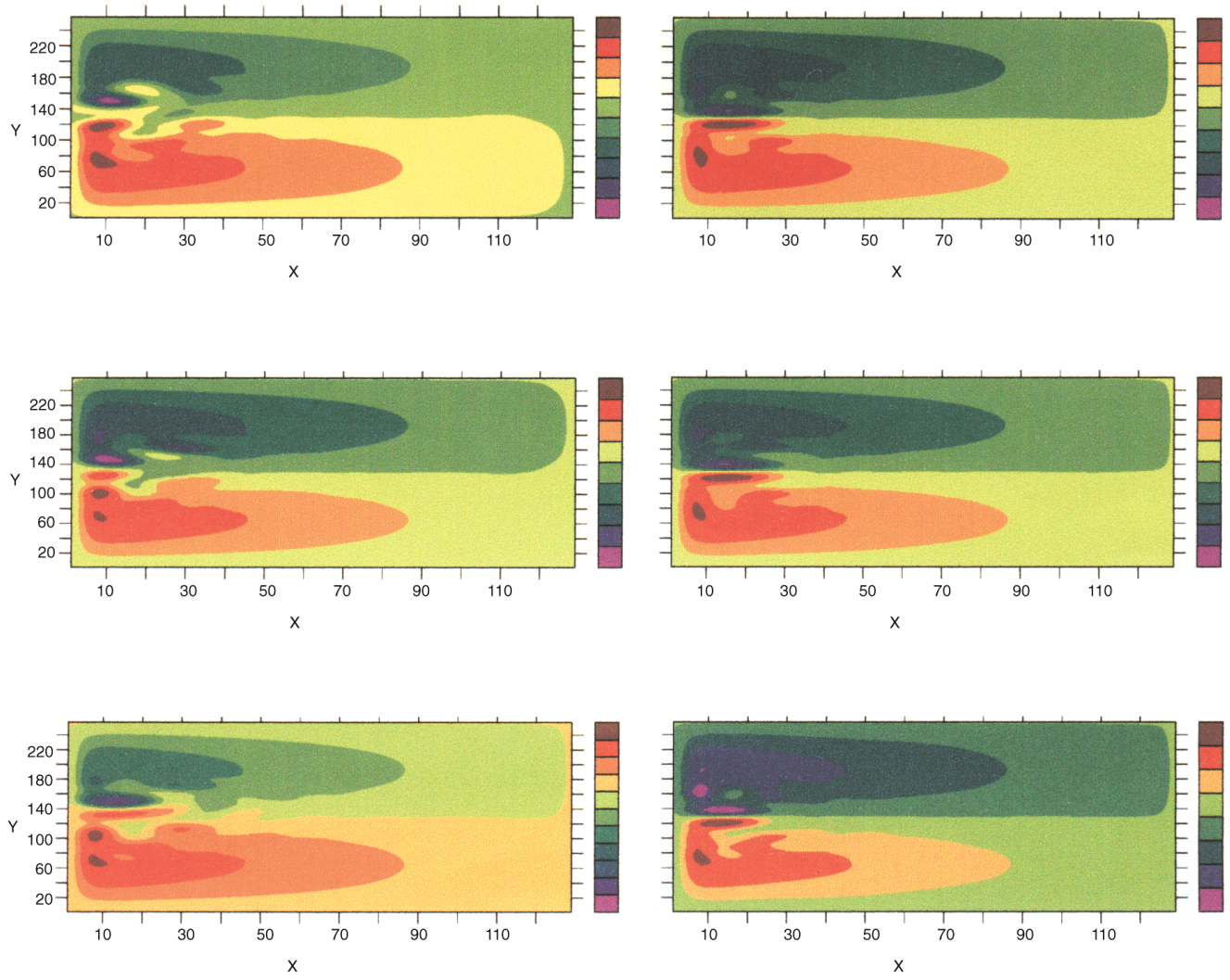
1. Streamfunction patterns for the low energy states (left panels) and high energy ones (right panels).
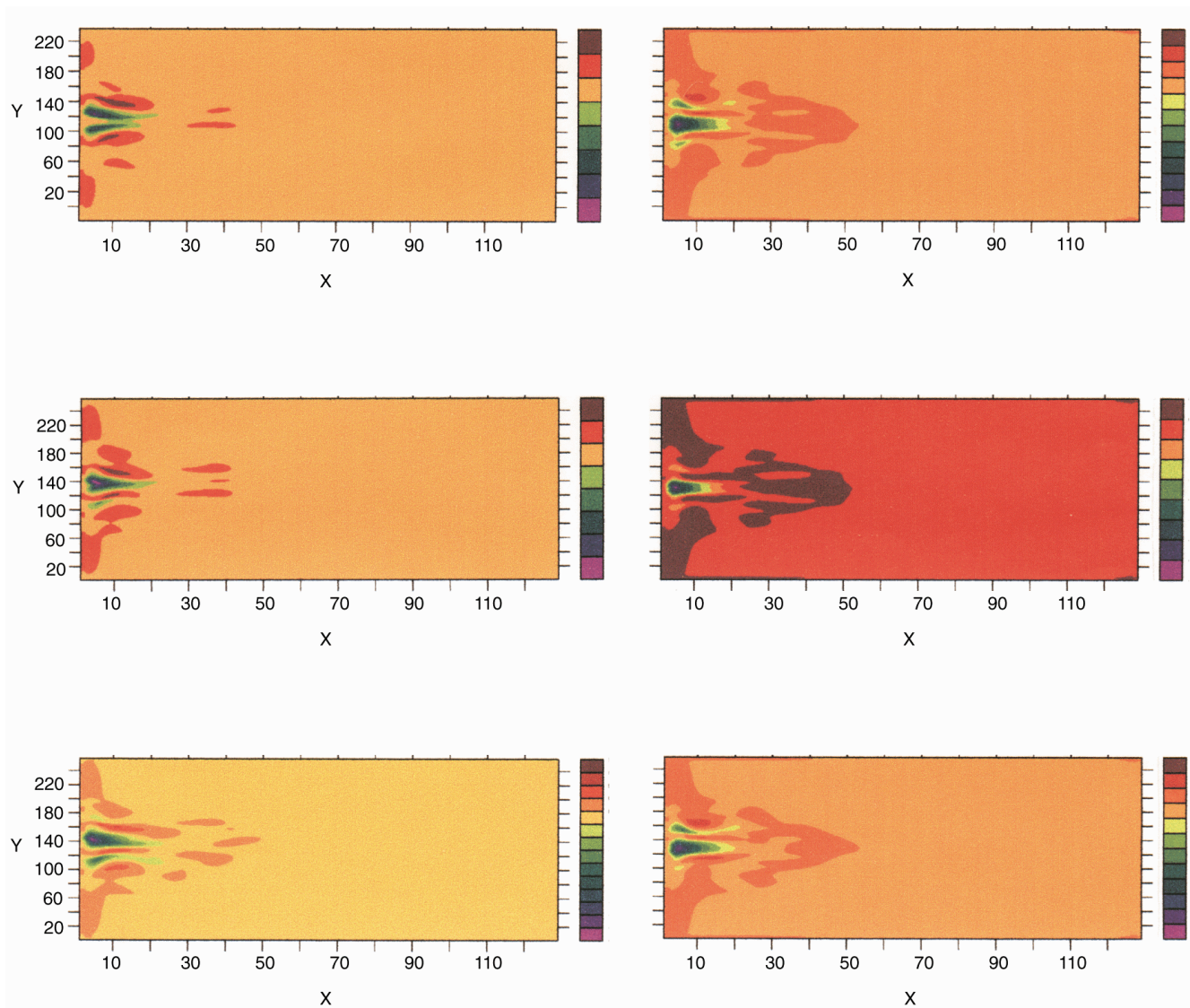
2. Dominant singular vectors for each of the streamfunction patterns of Fig. 1, in corresponding order.

REFERENCES

B.E. Farrell, 1990. Small error dynamics and the predictability of flows, Journal of Atmospheric Sciences, 47, 2409-2416.

B.F. Farrell and A.M. Moore, 1992. An adjoint method for obtaining the most rapidly growing perturbation to oceanic flows, Journal of Physical Oceanography, 22, 338-349.

A.M. Moore, 1991. Data assimilation in a quasi-geostrophic open ocean model of the Gulf Stream region using the adjoint method, Journal of Physical Oceanography, 21, 398-427.

A.M. Moore and B.F. Farrell, 1993. Rapid perturbation growth on spatially and temporally varying oceanic flows determined using an adjoint method: application to the Gulf Stream, Journal of Physical Oceanography, 23, 1682-1702.

PUBLICATIONS

Malanotte-Rizzoli, P., Meacham, S., Mahadevan, A., and Lu, J., 1999: The predictability of basin-scale wind-driven circulation, submitted.